SCIENTIA
FORESTALIS

Automated selective thinning via multicriteria maetaheuristic procedure

Desbastes seletivo automatizado via procedimento metaheurístico multicritério

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Resumo

O desbaste em povoamentos florestais é uma prática silvicultural importante para a regulação da competição entre indivíduos, bem como o restabelecimento do crescimento das árvores remanescentes de maior potencial de comercialização. Entretanto, a seleção dos indivíduos a serem removidos é um processo complexo que envolve a habilidade dos técnicos em identificar diversas variáveis no campo e tomar a decisão mais apropriada. Este processo pode obter respostas heterogêneas e sem padronização das equipes, podendo promover escolhas prejudiciais ao manejo da floresta. Desta forma, o estudo teve como objetivo aplicar o uso da metaheurística *simulated annealing* multicritério para a seleção das árvores a serem removidas no desbaste seletivo, considerando a estrutura diamétrica do povoamento, altura, ocupação do dossel, área de copa e qualidade da árvore. O estudo foi conduzido em um povoamento de *Eucalyptus grandis* contendo 336 árvores, sendo obtidas informações de DAP, Ht, área de copa, qualidade da árvore, volume e ocupação da copa. Foram simulados 10 cenários compreendendo diferentes pesos na função multicritério a ser minimizada. Os resultados mostraram que a metaheurística é capaz de selecionar indivíduos para o desbaste seletivo, apresentando vantagens de uma melhor padronização das escolhas, a partir de critérios pré estabelecidos, independente da probabilidade de seleção das árvores a nível do povoamento. Conclui-se que o uso desta ferramenta irá se tornar mais promissora após a utilização da tecnologia de aquisição de dados eficientes, porém sendo sempre um método de auxílio à tomada de decisão.

Palavras-chave: inteligência artificial, manejo florestal, tomada de decisão, eucalipto.

Abstract

Forest thinning is an important forestry practice for regulating competition among individuals, as well as for reestablishing the growth of the remaining trees with greater potential for commercialization. However, the selection of individuals to be removed is a complex process which involves technicians skilled in the identification of various variables in the field and making the most appropriate decision. By means of this process one can obtain heterogeneous responses and the lack of technical team standardization may lead to prejudicial choices for forest management. Therefore, the aim of the study was applying a multi-criteria simulated annealing metaheuristic for the selection of trees to be removed in selective thinning, considering the diametric structure of the forest stand, height, canopy occupation, crown projected area and the tree quality. The study was conducted on a forest stand of *Eucalyptus grandis* containing 336 trees. Ten scenarios were simulated, covering different weights in the multi-criteria function to be minimized. The results show that the metaheuristic is capable of selecting individuals for selective thinning, presenting advantages such as improved standardization of choices using predetermined criteria, independent of the probability of selecting trees at the stand level. It was concluded that the use of the method will become more promising after using the efficient data acquirement technology, while always being a method for decision making.

Keywords: artificial intelligence, forest management, decision making, eucalyptus.

INTRODUCTION

Thinning is a usual silvicultural practice designed to supply the market with high quality and dimension of logs. This activity is described as an operational process applied in a stand when some trees are cut off. As result, the space grows between the remaining trees and it enhances individual growth. Normally, the remaining trees are those which are dominant and co-dominant in the stand; they present a high growth rate, good crown formation, high-

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er wood quality and greater revenue potential. Dominated and suppressed trees are eliminated throughout the rotation to reduce the stand competition while generating interim revenues.

According to Tschieder et al. (2012) thinning reduces the variability range of the remaining trees, owing to selection, generating more efficient individual growth (ZEIDE, 2005; MCDOWELL, et al., 2007) due to less competition in the stand (LEWIS; FERGUSON, 1993; VANCLAY, 1994; WEST, 2006; PRETZSCH, 2009). The remaining trees, after thinning, may also undergo a pruning regime to increase their quality and market value, depending on the strategy adopted.

The choice of the remaining trees includes not only the threshold diameter size, but also a set of evaluations covering stem quality, spatial location of the tree, vitality and crown distribution. On the other hand, the decision making of the process should be direct and quick in the field. At this stage, some problems may arise in standardization and delay the operations.

Bohanec (2009) defines that decisions is a mental process involving the judgment of various alternatives and options, which generally present an order of preference established by the decision maker. It can be concluded that the thinning practice is not an exact science (DAUME; ROBERTSON, 2000).

Thinning practices have always accompanied the development of forest engineering (ZEIDE, 2001), as a dynamic theme that is updated over time. However, this practice is still realized following the technicians' individuality and their decisions, which could include errors.

The question arises is how to automate the selection process of trees to be thinned in a stand and following a technical criterion for its standardization. An answer or challenge of that can be expressed in two strategies: a) exhaustive training of the technician in the field or b) through artificial intelligence following a set of multiple criteria/rules. Thus, the objective of this study was to test a multi-criteria simulated annealing metaheuristic for the thinning. The selection of trees was based on the diametric structure of the stand, canopy occupation, crown projected area and tree quality.

MATERIAL AND METHODS

Data Base

The stand under study is located at the University of Lavras and it is part of a study on the effects of pruning trees submitted to different spacing, which was established in 2006. The plantation has a spacing of 3.0 x 5.0 meters and occupies an area of 0.58 ha. The species cultivated was *Eucalyptus grandis* presenting a total of 336 trees. The data were collected using a census by measuring the dbh (diameter at breast height = 1.3m from the ground). The nondestructive cubage was conducted on 28 selected trees using a hypsometer and Wheeler pentaprism. Diametric measurements were taken at various heights and the individual volume was taken using the Smalian method. Other variables were measured at an individual level: total height and crown projection.

Volumetric, hypsometric and projected canopy area models (Table 1) were adjusted using the least squares method in the R software (2011). The purpose was to estimate these variables for the entire stand (336 trees) and support the metaheuristic search.

Tabela 1. Mathematical models adjusted to estimate the variables at individual level tree.
Table 1. Modelos matemáticos ajustados para estimar as variáveis individuais das árvores.

Model type	Mathematical formula
Hypsometric (m)	$\ln(Ht_i) = \beta_0 + \beta_1\left(\frac{1}{D_i}\right) \pm \varepsilon_i$
Volumetric (m³)	$V_i = \beta_0 + \beta_1 D_i^2 .Ht_i \pm \varepsilon_i$
Projected crown area (m²)	$Ac_i = \beta_0 + \beta_1 D_i + \beta_1 D_i^2 \pm \varepsilon_i$

ln - natural logarithm; Ht – total height (m); D - diameter at 1.30 m from the ground (cm); V - volume (m³); Ac – projected crown area (m²); ε - estimation error; β_i - parameters of the model.

Furthermore, the Leaf Area Index (LAI) was obtained using the LAI 2200 device from Li-cor® as specified by the manufacturer (LI-COR, 2009), using a systematic scheme composed of 26 samplings spaced at 12 x 18 meters. The punctual information from the index was converted into a canopy occupation map through ordinary kriging using the least squares method in ArcGis 9.3. Through the localization of the trees and superimposition on the kriging map (Figure 1), the trees were classified into 5 canopy occupation classes. There was a further qualitative evaluation of the trees, involving the stem quality and its physical structure, covering three classes (I - high, II - average and III - low).

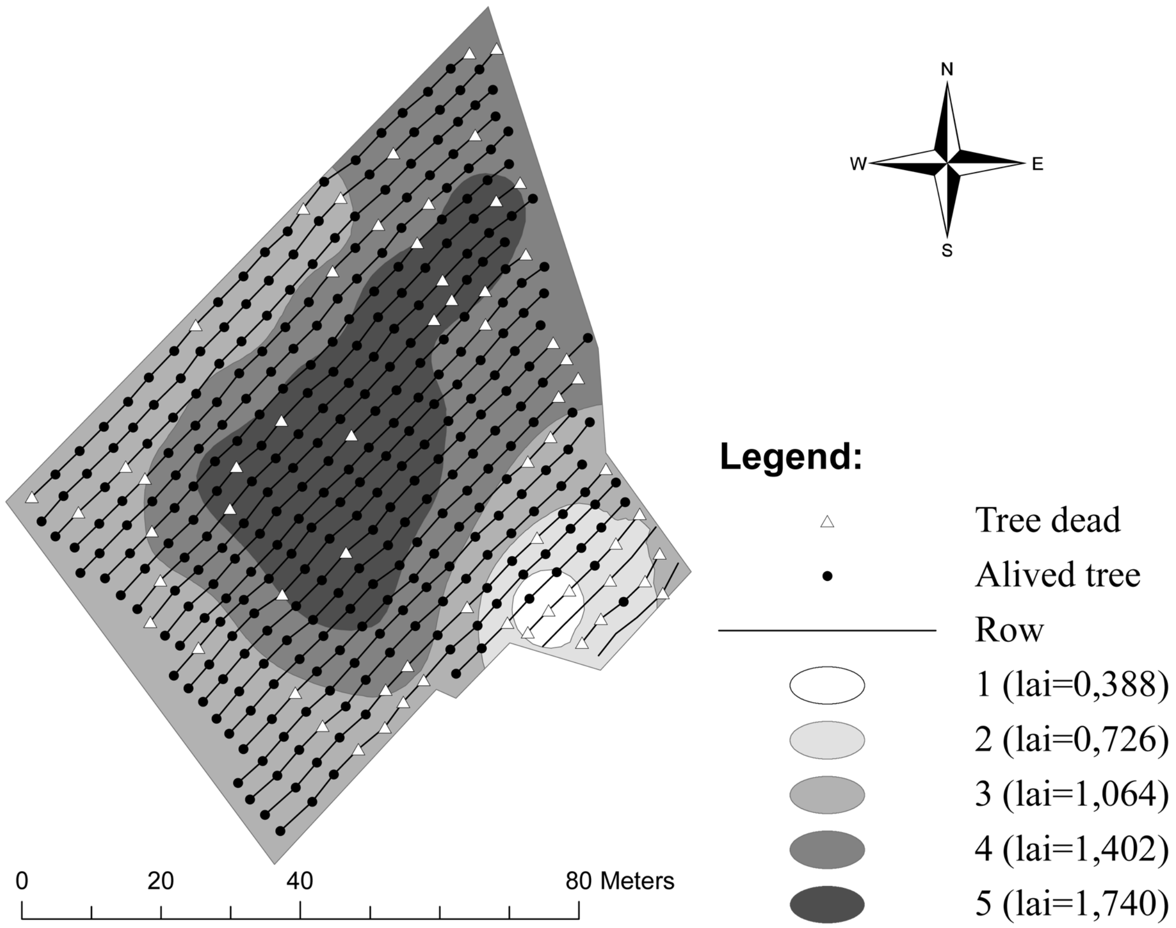

Figure 1. Spatial distribution of the trees in the stand considering the canopy occupation classes by ordinary kriging.
Figura 1. Distribuição espacial das árvores presentes no talhão considerando as classes de ocupação do dossel pela krigagem ordinária.

Multi-Criteria Metaheuristic

Simulated annealing (SA) was used to select the trees for thinning, and it was programmed according to Kirkpatrick et al. (1983). The implementation of the SA was undertaken using the programming language *Visual Basic*® version 6.0 *Enterprise* from *Microsoft*® and a computer with an Intel(R) Core(TM) i3-2100 3.10 Ghz processor and 8GB of RAM.

Thus, the function (1) was elaborated considering the multi-criteria problem covering a set of technical goals. In this case, quantitative and qualitative criteria based on diameter, height, crown projected area, and canopy occupation and tree quality were applied. It was established that all variables have the same importance and then the data were normalized into a scale (0-1). Weighting was used according to the scenarios tested. All tests were processed 100 times per option.

$$E(x) = w_1 \cdot \sum_{i=1}^{n} D_i + w_2 \cdot \sum_{i=1}^{n} H_i + w_3 \cdot \sum_{i=1}^{n} CA_i + w_4 \cdot \sum_{i=1}^{n} CP_i + w_5 \cdot \sum_{i=1}^{n} Q_i \quad (1)$$

Where: E(x) = multi-criteria function to be minimized and designated as energy; D_i = standardized dbh (0 - 1) belonging to tree *i*; H_i = standardized total height (0 - 1) belonging to tree *i*; CA_i = standardized crown area (0 - 1) belonging to tree *i*; CP_i = standardized canopy occupation level (0 to 1) for the geographical location of the tree *i* in the stand, considering the LAI; Q_i = standardized tree quality (0 - 1) belonging to tree *i*; W_j = weight associated to each criteria *j* {1, 2, 3, 4,and 5}, described above; n = total number of individuals.

The application of a metaheuristic requires previous analyses to ensure the reliability of the solution encountered (MIDDLETON, 2004). Therefore, tests were conducted considering the selection of trees with smaller diameters, where the number of selections includes two options: I- 50 trees (15.8% of the population) and II- 100 trees (29.7% of the population). However, only a diameter criterion (w_1=1) was used in this test. At this stage, the following metaheuristic configuration was taken into consideration: a) initial temperature of 5000; b) number of interactions equal to 15000; c) temperature reduction function (T): T=T0.9 and d) search for new solutions: 10% at each iteration. The results obtained by SA were compared with those obtained by two forest engineers at different periods of time.

Simulation of multi-criteria selective thinning scenarios

The decision making for the tree selection for thinning reflects the technical efforts of attending to a set of conflicting objective dilemmas; where this complexity requires qualified procedures for the decision. The choice of a single option will be made after exploring and evaluating some scenarios. Table 2 presents a set of 10 scenarios tested, covering 3 thinning intensities of the total number of trees as well as the adoption of different weights for each criterion. An increasing hierarchy of importance was used; i.e. the higher the value, the higher the importance. The best option was selected after 30 processes per scenario.

Tabela 2. Multi-criteria selective thinning scenarios on a stand of *Eucalyptus grandis*.
Table 2. Cenários multicritérios de desbaste seletivo para o talhão de *Eucalyptus grandis*.

Scenarios	Thinning intensity (%)	Weighting per criteria				
		w_1	w_2	w_3	w_4	w_5
A	30	1	1	1	1	1
B	30	1	0	0	1	1
C	30	0	1	0	1	1
D	30	0	0	1	1	1
E	30	1	0	0	2	3
F	30	2	0	0	1	3
G	40	1	0	0	2	3
H	40	2	0	0	1	3
I	50	1	0	0	2	3
J	50	2	0	0	1	3

w_1- weighting for the diameter criteria; w_2- weighting for the height criteria; w_3- weighting for the projected crown area criteria; w_4- weighting for the canopy occupation criteria and w_5- weighting for the tree quality criteria.

RESULTS AND DISCUSSION

Preliminary analysis of the tests

The results of the selection tests indicated an increase in the selection error percentage by the metaheuristic (Table 3) when the number of selected trees increased. However, the processing run time was similar between the selection options (I- 50 and II-100). The best solutions obtained for the two options remained close to the optimum and below 5%, with the optimum solution obtained by merely sorting the data in a descending order.

The mean percentages of individuals selected by diameter classes, after 100 process times, presented a natural tendency of reduction (negative exponential curve) when the classes increased (Figure 2). This tendency was independent of the number of trees to be selected (50 or 100). This fact positively proves the strategy adopted for the function E(x) present in the SA.

Usually, the reforestation forest presents a typical normal distribution of diameter, with a low frequency of individuals at the two ends of the distribution, which results in low selection probability (Figure 1). However, the metaheuristic managed to select the majority of the trees with lower diameters for the options tested. The effectiveness of the metaheuristic against the capacity of technicians work was superior, according to table 4. Another important point was the time spent by technicians in the field, as the large majority needed to return to lines already visited in order to reach the desired number of trees, which increases the final operation time. This situation can become more serious when increasing the stand area and the number of trees to be selected.

Tabela 4. The comparative error from SA and technicians work for 100 trees selected.
Table 4. Erro comparative entre SA e o trabalho dos técnicos para as 100 árvores selecionadas.

Selection method	Solution	100 trees	
		Error (%)	Time (s)
Multi-criteria metaheuristic	Best	4.68	261*
	Average	5.99	262*
	Worst	7.32	260*
Forest Engineers	1	12.43	2,520
	2	7.76	2,220

* considering only algorithm search time; and not collecting the data in the field.

Tabela 3. Result of the tree section test using the diameter criterion after 100 processing.
Table 3. Resultado do teste de seleção das árvores empregando o diâmetro como critério após 100 processamentos.

Statistics	50 trees			100 trees		
	E(x)	Error (%)	Time (s)	E(x)	Error (%)	Time (s)
Minimum	0.01787	0.49	236	0.04518	4.68	233
Average	0.01812	1.89	238	0.04576	6.01	255
Maximum	0.01831	2.93	242	0.04633	7.32	268

E(x) – energy; Error (%) – percentage deviation from optimum and Time (s) – computational run time in seconds.

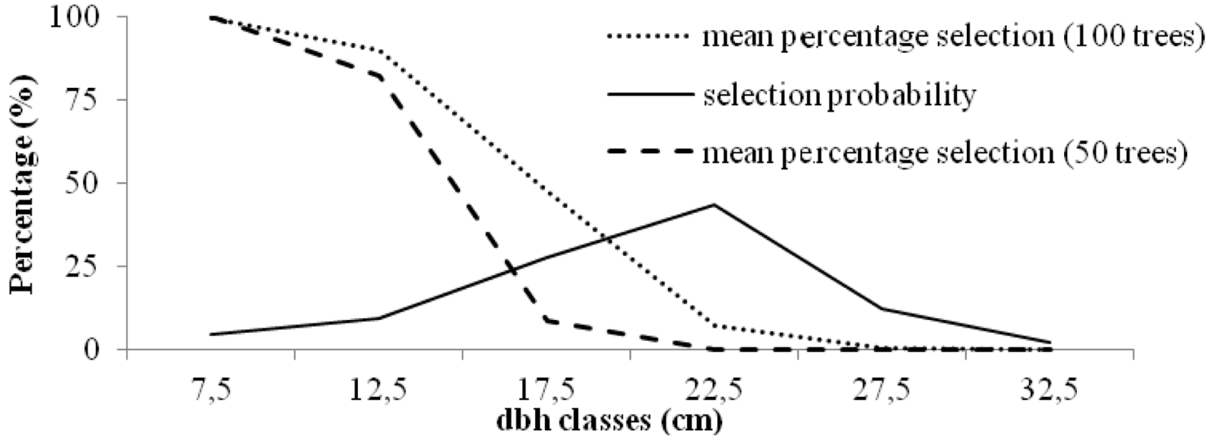

Figure 2. Relationship between the real selection probability and mean percentage selection by SA per diameter classes.
Figura 2. Relação entre a probabilidade real de seleção e a porcentagem média da seleção pela SA por classe de diâmetro.

Evaluation of the multi-criteria scenarios

The greatest criticism of the algorithms for selecting trees for thinning is that they do not exactly reflect what is applied by the technicians, owing to the criteria adopted, such as the diameter alone, for example (DAUME; ROBERTSON, 2000). Therefore, when incorporating variables such as competition among individuals and the quality of the tree, this allows for improvements in the decision aimed at overcoming failures in the algorithm. However, the threshold between the ideal and the applied could be economically unfeasible in large thinning programs, considering the current methods of collecting information.

On the other hand, the choice of one single variable such as dbh is a flaw that can be committed, as the elimination of smaller individuals does not always guarantee a reduction in competition and maximization of individual growth of the remaining trees, as observed in Zeide (2001). However, when testing the behavior of the algorithm to solve a problem, the choice of a simple situation which can be controlled becomes appropriate. The results of the sceneries are shown in Table 5.

In general, tree selection using the algorithm concentrated on the worst classes of each criterion, confirming the operation of the proposed method. According to the results of scenario A, 81% of trees belonged to the diameter classes from 5 to 20 cm, 85% in the crown projected area class varying from 0.44 to 9.6 m^2, 82% in the three largest classes of canopy occupation (3, 4 and 5) and more than 93% for the trees with the worst quality in the stand. Nevertheless, only 19% of the trees were categorized in the height class of 7.47 to 17.67 meters. There was no problem or failure in the method because the height mean was 25.63 meters in the stand.

Tabela 5. Result of the scenarios and their percentages of tree selected per class of occurrence.
Table 5. Resultado dos cenários e suas porcentagens de árvores selecionada por classe de ocorrência.

Scenarios	Criteria											Volume removed (m³)
				4					5			
	1*	2**	3***	Classes					Classes			
				1	2	3	4	5	1	2	3	
A	81.0	19.0	85.0	15.0	3.0	21.0	30.0	31.0	7.0	23.0	70.0	21.174
B	69.0	4.0	55.0	0.0	3.0	28.0	34.0	35.0	4.0	30.0	66.0	27.455
C	67.0	10.0	56.0	6.0	2.0	28.0	29.0	35.0	2.0	28.0	70.0	26.868
D	67.0	5.0	53.0	1.0	2.0	27.0	31.0	39.0	5.0	26.0	69.0	27.972
E	67.0	7.0	55.0	4.0	2.0	24.0	33.0	37.0	1.0	31.0	68.0	27.826
F	67.0	11.0	58.0	11.0	3.0	28.0	30.0	28.0	0.0	28.0	72.0	25.967
G	57.5	8.2	47.8	6.7	2.2	28.4	30.6	32.1	6.7	35.1	58.2	40.739
H	67.2	9.0	55.2	13.4	3.0	26.1	27.6	29.9	5.2	27.6	67.2	37.953
I	54.8	5.4	44.0	6.0	2.4	31.5	31.0	29.2	16.7	32.1	51.2	54.547
J	60.7	9.5	50.6	11.9	3.6	24.4	31.0	29.2	16.1	30.4	53.6	49.665

Percentage values including only the first three classes of each criteria, namely: * diameter classes (cm) with an interval from 5 to 20; ** height class (m) with an interval of 7.47 to 17.67 and *** projected crown area class (m2) with an interval from 0.44 to 9.62.

The crown projected area and the height of the trees were derived from regression models based only on the dbh variable. Thus, these criteria established a high correlation among them for selecting the same trees for thinning. This relationship covered values of 72% (scenarios B and C), 71% (scenarios B and D) and 68% (scenarios C and D).

Analyzing the relationship between the scenarios it was found that there is a distinction and effect of the weightings of trees in the tree selection by the algorithm, a fact proven when observing that only 39% of the trees selected belonged to the first four scenarios (A, B, C and D). Introducing the scenarios E and F into the previous set, this value reaches 29%. The neutral scenario (A) which has equal weightings (w_i = 1) among criteria, was capable of selecting 93% of the trees in the worst quality class (2 and 3), increasing to 99% (scenario E) and 100% (scenario F) in the scenarios with a higher weighting for this criterion (w_i = 3). The largest classes of canopy occupation (3, 4 and 5) followed the same tendency, with 82% in scenario A (w_i = 1) and 94% for scenario E (w_i = 3). The same behavior was not observed for the dbh criteria among scenarios A, E and F.

When increasing the number of trees to be removed (30%, 40% and 50%) and fixing the weighting of the criteria, two groups of scenarios are established – the first being A, G and I and the second F, H and J. The results in the selection of the trees demonstrates that these increases promote greater difficulty for the algorithm in selecting the same trees between the scenarios in these groups, noting in the first group: 81% (E and G), 67.16% (E and I) and 66.67% (G and I), and in the second group: 81% (F and H), 64.18% (F and J) and 64.29% (H and J). In total, 21 trees were selected simultaneously for these two groups, representing around 6.25% of the population.

Paudyal and Majid (1990) used the diameter and the crown area to recommend thinning in commercial plantations of *Acacia mangium* in Malaysia, whose criteria indirectly reflect the quality of the trees and level of competition suffered and which are the same criteria tests in 9 scenarios in the present study. The elimination of individuals with low tree quality and growth rates, as well as open canopy space, favors high yield and crown development.

A reduced number of variables input into the multi-criteria model can generate imprecise results, for example the dbh alone, as various variables are evaluated by the technicians when selecting trees. These details are better observed in the work of Kahn (1995) and Daume and Robertson (2000).

McDowell et al. (2007) commented that the LAI does not establish a correlation with growth in the basal area, as it is dependent on factors such as water and nutrient availability. However, the index reflects the canopy occupation and it may perfectly represent a possible competition level and become an important criterion. The spatialization, via geostatistics, permits a better understanding of this important variable in the stand, as introduced in all of the scenarios tested. It is expected that selective thinning including this variable should be capable of identifying a higher canopy occupation at certzin points, thereby facilitating the choice of removal through thinning.

The increased number of selected trees is a factor that compromises the quality of the solution generated. Consequently, it increases the algorithm searches generating a larger number of combinations to be analyzed. According to Vanclay (1994), tree selection for thinning becomes more complex than the mathematical modeling, when the number of remaining/explored trees is defined previously. This statement is strengthened when it is perceived that the criteria may be changed in the selection process, altering the final result generated. It is further seen that the use of stochastic over deterministic procedures becomes preferable, as they are closer to human reasoning in resolving problems of this nature.

The occurrence of new solutions is possible after processing the same scenario, due to a stochastic behavior of the metaheuristic, and is perhaps a negative point. A similar problem was obtained by Minowa (2005) when using the machine learning system (C4.5) in the tree selection for thinning.

Algorithms will never replace technicians, but can support them in standardizing decision making in their choices. Furthermore, the number of technicians trained for the appropriate execution of selective thinning is rudimentary and inefficient (MINOWA, 2008). On the other hand, there is a high number of combinations of criteria that a technician needs to evaluate per tree, and so two or more similar trees may not be equally recommended for the selective thinning.

CONCLUSION

The use of a multi-criteria metaheuristic procedure for the tree selection for thinning depends on the quantification of a set of variables via a census. This ensures the viability and quality of the solution generated, i.e. elimination of the trees which really need to leave the system. The difference between the metaheuristic and the technician lies in the standardization of the process, allowing the criteria desired by managers to be attained as far as possible. A positive point of the process is the possibility of producing maps indicating which trees should really be removed, reducing the possibility of errors in the field. The development of laser technologies enables to measure variables faster and could make the process feasible and automatable, according to a cost/benefit relationship. The tests undertaken showed that there is a possibility of using artificial intelligence in the standardization and automation of the operation, while remaining attentive to the quantification of the weighting given to each criterion. Furthermore, there is a need to introduce criteria that avoid the high number of neighboring trees to be selected for thinning.

ACKNOWLEDGEMENT

We would like to thank all of the students and professors that participated, from the planting of the area to the data collection, especially all students: Matheus, Jennifer, Vinicius, Paulo Henrique, Kallil and Marcela.

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Recebido em 27/05/2013
Aceito para publicação em 16/04/2014